Chapter 8 - Day 3

Riemann Sum

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{k=1}^{n} f(p_k)\,\Delta x_k$$

where $f(x)$ is continuous on $[a,b]$, partitioned into n subintervals. The k^{th} interval contains point p_k and has width Δx_k

Ex: Suppose we want to estimate

$\int_1^5 8^x \, dx$ by evaluating $\sum_{k=1}^{n} 8^{1+k\Delta x} \cdot \Delta x$

if $\Delta x = .2$, what is n?

Δx is width of subintervals

$[1,5]$ has n subintervals of length $.2$

$$\Delta x = \frac{5-1}{n} = .2$$

$$\frac{4}{n} = .2$$

$$4 = .2n$$

$$\frac{4}{.2} = n$$

$$\boxed{n = 20}$$

Ex: Estimate $\int_4^{10} x^2\,dx =$

$$\sum_{k=1}^{n} (2+k\cdot\Delta x)^2 \cdot \Delta x$$

If $n=10$, what is Δx?

[4,10] is split into 10 subintervals

$$\Delta x = \frac{10-4}{10} = \frac{6}{10} = \boxed{.6}$$

Ex: We will estimate $\int_{-6}^{0} x^2 dx$ by the sum $\sum_{k=1}^{n} \left[A + B(k\Delta x) + C(k\Delta x)^2\right] \cdot \Delta x$

where $n = 30$ and $\Delta x = .2$

find A, B, C.

$\int_{-6}^{0} x^2 \, dx$ tells us $a = -6$, $b = 0$, $f(x) = x^2$

then our Riemann Sum is...

right endpoints

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} f(-6 + k\Delta x) \cdot \Delta x$$

$$= \sum_{k=1}^{n} (-6 + k\Delta x)^2 \cdot \Delta x$$

$$= \sum_{k=1}^{n} (36 - 12k\Delta x + (k\Delta x)^2) \cdot \Delta x$$

then we'll play the matching game!

$$\boxed{A = 36 \quad B = -12 \quad C = 1}$$

Ex: You estimate $\int_3^{15} f(x)\,dx =$

$$\sum_{k=1}^{n} f(\underbrace{3+k\cdot\frac{A}{n}}_{\text{right endpoint}})\cdot \frac{A}{n} \leftarrow \Delta x$$

What is A?

$a=3$, $b=15$

$$\Delta x = \frac{15-3}{n} = \frac{12}{n}$$

then $x_k = a + k\Delta x$

$$= 3 + k\cdot\frac{12}{n}$$

thus $\boxed{A=12}$

Ex: Estimate the area under the graph of $f(x) = x^3$ from $x = 6$ to $x = 36$ by Partitioning $[6, 36]$ into 30 subintervals, using right endpoints. What is the area of the 20th rectangle?

$$\Delta x = \frac{36-6}{30} = \frac{30}{30} = 1$$

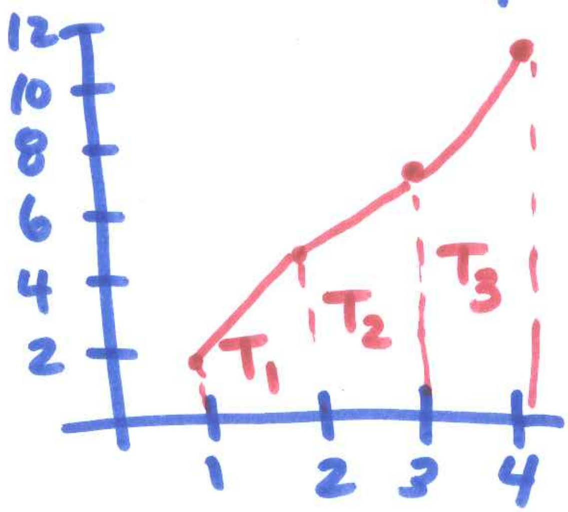
$$x_k = a + k\Delta x$$
$$= 6 + k$$

$$A(\square) = f(x_{20}) \cdot \Delta x_{20}$$
$$= f(6+20) \cdot 1$$
$$= 26^3 \cdot 1$$
$$= \boxed{17,576}$$

Ex: Suppose we are given data points for a function $f(x)$:

x	1	2	3	4
$f(x)$	2	5	8	12

if f is linear function between the given points, find $\int_1^4 f(x)\,dx$.



$$\int_1^4 f(x)\,dx = T_1 + T_2 + T_3$$

$$= \frac{(2+5)}{2} + \frac{(5+8)}{2} + \frac{(8+12)}{2}$$

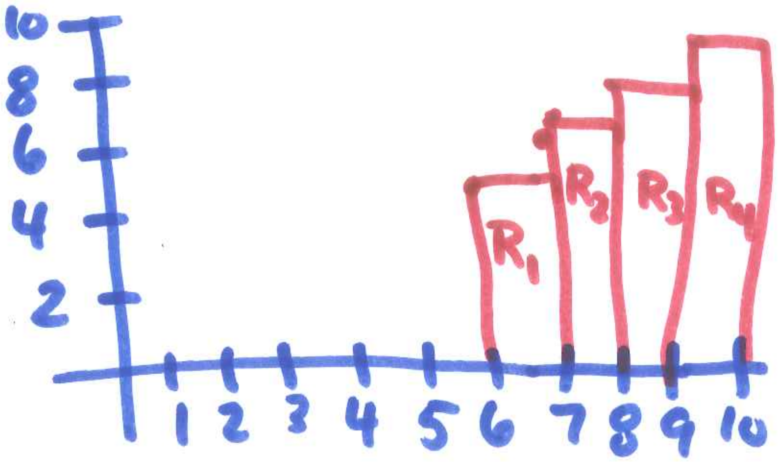
$$= 3.5 + 6.5 + 10$$

$$= \boxed{20}$$

Ex: let $f(x)$ be the greatest integer function.

(Thus $f(2.3)=2$, $f(4)=4$, $f(6.9)=6$)

Find $\int_6^{10} f(x)\,dx$



$$\int_6^{10} f(x)\,dx = R_1 + R_2 + R_3 + R_4$$

$$= f(6)\cdot 1 + f(7)\cdot 1 + f(8)\cdot 1 + f(9)\cdot 1$$

$$= 6 + 7 + 8 + 9$$

$$= \boxed{30}$$